Mathematical Modeling, Energy Consumption and Thin Layer Drying Kinetics of Carrot Slices Under Microwave Oven

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ABSTRACT: The thin-layer drying characteristics of carrot slices were investigated under four microwave powers; 100, 300, 500 and 700 W and slice thickness of 7 mm. Data were analyzed to obtain diffusivity values from the period of falling drying rate. Five mathematical models for describing the thin-layer drying behavior of carrot were investigated. The results show that the Midilli et al. is the most appropriate model for drying behavior of thin layer carrot slices. An analysis of variance (ANOVA) revealed that microwave power significantly affected the drying time, drying rate, effective diffusivity and specific energy consumption. The effective diffusivity varied from 4.23×10⁻⁷ to 7.34×10⁻⁶ m²/s, and the activation energy was determined to be 92.33 W/g. Specific energy consumption values ranged 10.27 to 23.29 kW·h/kg water and the optimized specific energy consumption was obtained 300 W microwave levels.

Keywords: Carrot Slices, Energy Consumption, Microwave Drying, Effective Diffusivity, Modeling.

INTRODUCTION

Drying is the process of removing the moisture in the product up to certain threshold value by evaporation. In this way, the product can be stored for a long period, since the activities of the microorganisms, enzymes or ferments in the material are suppressed via drying (Ozkan et al, 2007). In particular, convective hot-air drying is extensively employed as a preservation technique. The major draw-back of convective hot-air drying method, from an energy point of view, is the longer drying period, higher drying temperature and therefore high energy consumption, which may be as high as 6000 kJ/kg of water evaporated (Al-Harahsheh et al, 2009). In general, energy efficiency in drying is closely related to drying times. Microwave is an electromagnetic wave in the frequency range of 300–30000 MHz. The conversion of microwave energy into heat in the food is because of the presence of water. The quick absorption of energy by water molecules causes rapid evaporation of water, resulting in high drying rates of the food. Compared to hot air drying, microwave or hybrid/combined microwave drying techniques can greatly reduce the drying time (up to 50%) of biological materials without quality degradation, therefore microwave method offers significant energy savings (Soysal et al, 2006; Beaudry et al, 2003). The most relevant aspects of drying technology are the mathematical modeling of the process and the experimental setup (Akpinar et al, 2006). The modeling is basically based on the design of a set of equations to describe the system as accurately as possible. Drying characteristics of the particular products being dried and simulation models are needed in the design, construction and operation of drying systems (Celma et al, 2011). The aim of this research was (i) to determine the influence of microwave power on the energy consumption and drying kinetics during microwave dehydration of carrot slices and (ii) to fit the experimental moisture data to five mathematical models.

MATERIALS AND METHODS

Carrot samples were procured from local vegetable market in Tehran, Iran. The samples were stored at 4±0.5 °C before they were used in experiments. Carrots were washed under running water to remove the adhering impurities, and thinly sliced in thicknesses of 7 mm using a sharp stainless steel knife. The average initial
moisture content of the samples were found to be 87±0.1% wet basis, as determined by using convective oven at 105 °C for 24h. A domestic microwave oven (M945, Samsung Electronics Ins) with maximum output of 1000 W at 2450MHz was used for drying experiments. The dimensions of the microwave cavity were 327×370×207 mm. The oven has a fan for air flow in drying chamber and cooling of magnetron. The moisture from drying chamber was removed with this fan by passing it through the openings on the right side of the oven wall to the outer atmosphere. The microwave dryer was operated by a control terminal which could control both microwave power level and emission time. Experiments were performed at four microwave powers of 100, 300, 500 and 700 W. The moisture losses of samples were recorded at 30s intervals during the drying process by a digital balance (GF-600, A & D, Japan) and an accuracy of ±0.01 g. For measuring the weight of the sample during experimentation, the tray with sample was taken out of the drying chamber, weighed on the digital top pan balance and placed back into the chamber. Drying was carried out until the final moisture content reaches to a level less than 6% (w.b.). The experimental drying data were used to calculate the moisture ratio (MR) and drying rate (DR) using the following equations:

\[
MR = \frac{M_t - M_e}{M_0 - M_e}
\]

(1)

\[
DR = \frac{M_{t+dt} - M_t}{dt}
\]

(2)

where \(M_t\), \(M_0\) and \(M_e\) are the moisture content at any time of drying; initial moisture content and equilibrium moisture content (kg water/kg dm), respectively, DR is the drying rate (kg water/kg dm.min), \(M_{t+dt}\) is the moisture content at \(t+dt\) (kg water/kg dm), and \(t\) is drying time (min). Since the values of \(M_e\) are very small compared to \(M_t\) or \(M_0\), Eq. (1) can be simplified to \(M_t/M_0\) (Sarimeseli, 2011).

Five well-known thin-layer drying models in Table 1 were tested to select the best model for describing the drying curve of the carrot slices. The terms used to evaluate goodness of fit of the tested models to the experimental data were the coefficient of determination (\(R^2\)); root mean square error (RMSE) and the reduced chi-square (\(\chi^2\)) between the experimental and predicted moisture ratio values. Statistical values are defined as follows:

\[
\chi^2 = \sum_{i=1}^{N} \left(\frac{MR_{exp,i} - MR_{pre,i}}{N - z}\right)^2
\]

(3)

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{N} \left(\frac{MR_{exp,i} - MR_{pre,i}}{N}\right)^2}{N}}
\]

(4)

In these equations, \(N\) is the number of observations, \(z\) is the number of constants, \(MR_{exp}\) and \(MR_{pre}\) are the experimental and predicted moisture ratios, respectively.

### Table 1. Thin-layer drying models

<table>
<thead>
<tr>
<th>Model name</th>
<th>Model</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Page</td>
<td>(MR = \exp(-kt^n))</td>
<td>(Sarimeseli, 2011)</td>
</tr>
<tr>
<td>Newton</td>
<td>(MR = \exp(-kt))</td>
<td>(Soysal et al, 2006)</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>(MR = a \exp(-kt) + b)</td>
<td>(Akpinar et al, 2006)</td>
</tr>
<tr>
<td>Henderson and Pabis</td>
<td>(MR = a \exp(-kt)) + bt</td>
<td>(Soysal et al, 2006)</td>
</tr>
<tr>
<td>Midilli et al.</td>
<td>(MR = a \exp(-kt^n) + bt)</td>
<td>(Ozbek and Dadali, 2007)</td>
</tr>
</tbody>
</table>

*where, \(k\) is the drying constant and \(a, b, n\) are equation constants

During the drying process, diffusivity is assumed to be the only physical mechanism for the transfer of water to material surface and can be defined by Fick’s second law of diffusion for a slab as follows:

\[
\frac{\partial M}{\partial t} = D_{eff} \frac{\partial^2 M}{\partial x^2}
\]

(5)

By using appropriate initial and boundary conditions, Crank (Crank, 1975) gave the analytical solutions for various geometries and the solution for spherical object with constant diffusivity as follow:

\[
MR = \frac{8}{\pi^2} \sum \frac{1}{(2n+1)^2} \exp \left( - \frac{(2n+1)^2 \pi^2 D_{eff}}{4L^2} t \right)
\]

(6)
where \( D_{\text{eff}} \) is the effective diffusivity \( (m^2/s) \), and \( L \) is the half thickness of slab \( (m) \). For long drying times, only the first term \( (n=0) \) in the series expansion of the above equation can give good estimate of the solution, which is expressed in the logarithmic forms as follow:

\[
\ln(\text{MR}) = \ln\left(\frac{8}{\pi^2}\right) - \left(\frac{\pi^2 D_{\text{eff}}}{4L^2}\right) t \tag{7}
\]

The diffusion coefficients are typically determined by plotting experimental drying data in terms of \( \ln(\text{MR}) \) versus drying time \( (t) \), because the plot gives a straight line with a slope as \( \pi^2 D_{\text{eff}}/4L^2 \).

Specific energy consumption \( (E_s) \) of the drying process was expressed in \( \text{kW} \times \text{h/kg water evaporated} \). Specific energy consumption \( (E_s) \) of the drying process was expressed in \( \text{kW} \times \text{h/kg water evaporated} \). Therefore, the \( E_s \) could be determined as follows (Soysal, 2004):

\[
E_s = \frac{Pt}{m_w} \tag{8}
\]

where \( P \) is the microwave power input \( (W) \); and \( m_w \) is the mass of water evaporated \( (kg) \).

Inasmuch as temperature is not precisely measurable inside the microwave dryer, the activation energy is found as modified from the revised Arrhenius equation. In this method it is assumed as related to effective diffusion coefficient and the ratio of microwave output power to sample weight \( (m/p) \) instead of to air temperature. Then Equation (9) can be effectively used (Ozbek and Dadali, 2007) as follows:

\[
D_{\text{eff}} = D_0 \exp\left(-\frac{E_a m}{P}\right) \tag{9}
\]

where \( E_a \) is the activation energy \( (\text{W/g}) \), \( m \) is the mass of raw sample \( (g) \), and \( D_0 \) is the pre-exponential factor \( (m^2/s) \). All measurements were carried out in triplicate. ANOVA test was performed in order to examine the effect of microwave power on drying kinetics and energy consumption. The SPSS version 18.0 was used for statistical investigations. For all statistical analysis, the level of significance is fixed at 95%. Each factor having a \( P \) value \( \leq 0.05 \) was considered significant.

![Figure 1. Variation of moisture ratio with drying time for carrot slices](image)

**RESULTS AND DISCUSSION**

The moisture ratios versus drying time for the carrot slices at the selected powers are shown in Fig. 1. The total drying times to reach the final moisture content for the carrot sample were 52, 15, 10 and 9 min at 100, 300, 500, and 700 W, respectively. Obviously, within a certain microwave power range (100-700 W in this study), increasing output power speeds up the drying process, thus short the drying time (up to 83%). As seen in Figs. 2 and 3, all curves have two stages. The drying rate rapidly increases and then slowly decreases as drying progresses. The moisture content of the material was very high during the initial phase of the drying which resulted in a higher absorption of microwave power and higher drying rates due to the higher moisture diffusion. As the drying progressed, the loss of moisture in the product caused a decrease in the absorption of
microwave power and resulted in a fall in the drying rate. Similar findings were reported in the previous studies (Soysal et al., 2006; Wang et al., 2007; Therdtai and Zhou, 2009).

The drying rate by the microwave method can be described by Eq. (10):

\[
DR = At^u + \frac{ht}{1 + \exp\left(t^d\right)}
\]  

(10)

Parameters A, u, h and d of Eq. (10) are given in Table 2.

<table>
<thead>
<tr>
<th>P(W)</th>
<th>A</th>
<th>u</th>
<th>h</th>
<th>d</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>0.188</td>
<td>0.336</td>
<td>19.94</td>
<td>1.834</td>
<td>0.998</td>
</tr>
<tr>
<td>500</td>
<td>0.158</td>
<td>0.446</td>
<td>16.81</td>
<td>1.821</td>
<td>0.980</td>
</tr>
<tr>
<td>300</td>
<td>0.139</td>
<td>0.522</td>
<td>10.97</td>
<td>1.354</td>
<td>0.945</td>
</tr>
<tr>
<td>100</td>
<td>0.045</td>
<td>0.751</td>
<td>3.402</td>
<td>0.803</td>
<td>0.917</td>
</tr>
</tbody>
</table>

In order to take into account the effect of microwave power on the constant and coefficients of the Eq. (10), namely, A, u, h and d, the regression analysis was used to set up the relations between these parameters and the microwave power. Thus, the regression equations of these parameters against microwave power and
the drying rate model are as follows:

\[ h = 0.0278P + 1.6897 \quad R^2 = 0.969 \]
\[ u = 64.431P^{0.84} \quad R^2 = 0.983 \]
\[ d = -3\times10^{-6}P^2 + 0.0045P + 0.3711 \quad R^2 = 0.990 \]
\[ A = 2\times10^{-9}P^3 - 3\times10^{-6}P^2 + 0.0012P - 0.0564 \quad R^2 = 0.999 \]

The statistical results from the models are summarized in Table 3. The best model describing the thin-layer drying characteristics of carrot slices was chosen which has the highest \( R^2 \) values and the lowest \( \chi^2 \) and RMSE values. The statistical parameter estimations showed that \( R^2 \), \( \chi^2 \) and RMSE values were ranged from 0.9801 to 0.9998, 0.00002 to 0.0006, and 0.00435 to 0.031, respectively. Among of all the tested models, the Midilli et al. model gives the highest value of \( R^2 \) and the lowest values of \( \chi^2 \) and RMSE. It was determined that the value of the drying rate constant \( (k) \) increased with the increase in microwave powers. This implies that with increasing in microwave power drying curve becomes steeper indicating increase in drying rate.

<table>
<thead>
<tr>
<th>Model</th>
<th>Model Constants</th>
<th>( R^2 )</th>
<th>( \chi^2 )</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newton</td>
<td>( k=0.1405 )</td>
<td>0.9801</td>
<td>5.9\times10^{-7}</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>( k=0.0538 )</td>
<td>0.9924</td>
<td>5.9\times10^{-6}</td>
<td>0.02332</td>
</tr>
<tr>
<td></td>
<td>( k=0.321 )</td>
<td>0.9971</td>
<td>2.29\times10^{-4}</td>
<td>0.02447</td>
</tr>
<tr>
<td></td>
<td>( k=0.2281, n=0.7785 )</td>
<td>0.9985</td>
<td>2.6\times10^{-6}</td>
<td>0.00862</td>
</tr>
<tr>
<td></td>
<td>( k=0.606, n=0.8064 )</td>
<td>0.9996</td>
<td>2.6\times10^{-6}</td>
<td>0.00528</td>
</tr>
<tr>
<td>Page</td>
<td>( k=1.055, n=0.7439 )</td>
<td>0.9993</td>
<td>6.12\times10^{-5}</td>
<td>0.00825</td>
</tr>
<tr>
<td></td>
<td>( k=1.398, n=0.7126 )</td>
<td>0.9998</td>
<td>2.36\times10^{-5}</td>
<td>0.00825</td>
</tr>
<tr>
<td></td>
<td>( a=1.041, k=0.1272 )</td>
<td>0.9863</td>
<td>4.83\times10^{-4}</td>
<td>0.02594</td>
</tr>
<tr>
<td>Henderson and Pabis</td>
<td>( a=0.9716, k=0.4891 )</td>
<td>0.9933</td>
<td>4.83\times10^{-4}</td>
<td>0.02276</td>
</tr>
<tr>
<td></td>
<td>( a=0.9881, k=0.9292 )</td>
<td>0.9935</td>
<td>5.84\times10^{-4}</td>
<td>0.0254</td>
</tr>
<tr>
<td></td>
<td>( a=0.997, k=1.309 )</td>
<td>0.9971</td>
<td>2.78\times10^{-4}</td>
<td>0.0254</td>
</tr>
<tr>
<td></td>
<td>( k=0.1456, a=1.06, b=0.02581 )</td>
<td>0.9939</td>
<td>2.83\times10^{-4}</td>
<td>0.0175</td>
</tr>
<tr>
<td></td>
<td>( k=0.5274, a=0.9611, b=0.0197 )</td>
<td>0.9961</td>
<td>2.83\times10^{-4}</td>
<td>0.01808</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>( k=0.9762, a=0.9777, b=0.01335 )</td>
<td>0.9947</td>
<td>4.72\times10^{-4}</td>
<td>0.02431</td>
</tr>
<tr>
<td></td>
<td>( k=1.377, a=0.9845, b=0.01379 )</td>
<td>0.9985</td>
<td>1.4\times10^{-5}</td>
<td>0.02431</td>
</tr>
<tr>
<td></td>
<td>( k=0.5585, a=1.745, b=0.0016679, n=0.685 )</td>
<td>0.9989</td>
<td>5.1\times10^{-5}</td>
<td>0.00435</td>
</tr>
<tr>
<td>Midilli et al.</td>
<td>( k=0.6158, a=1.003, b=0.00389, n=0.726 )</td>
<td>0.9996</td>
<td>2.54\times10^{-5}</td>
<td>0.00563</td>
</tr>
<tr>
<td></td>
<td>( k=1.054, a=0.9994, b=0.0043, n=0.7443 )</td>
<td>0.9995</td>
<td>4.72\times10^{-5}</td>
<td>0.00821</td>
</tr>
<tr>
<td></td>
<td>( k=1.397, a=1.006, b=0.0086, n=0.7669 )</td>
<td>0.9998</td>
<td>2.09\times10^{-5}</td>
<td>0.00821</td>
</tr>
</tbody>
</table>

Figure 4 compares experimental data with those predicted with the Midilli et al. model for carrot slice samples at different microwave powers. The prediction using the model showed MR values banded along the straight line, which showed the suitability of these models in describing drying characteristics of carrot slices.

The effective moisture diffusivities of carrot slices for different microwave powers are presented in Table 4. The values ranged from \( 4.23\times10^{-7} \) to \( 7.34\times10^{-6} \) m\(^2\)/s. In Table 4, it was noted that \( D_{eff} \) increased progressively with the increase of drying microwave power. This might be explained by the increased heating energy, which would increase the activity of the water molecules leading to higher moisture diffusivity when samples were dried at higher microwave power. The values for \( D_{eff} \) obtained from this study lie within the general range \( 10^{-11} \) to \( 10^{-6} \) m\(^2\)/s for drying of food materials (Sacilik et al., 2006; Lee and Zuo, 2011). The values of \( D_{eff} \) are comparable with the reported values of \( 1.0465\times10^{-8} \) to \( 9.1537\times10^{-8} \) m\(^2\)/s mentioned for apple pomace microwave drying (Wang et al., 2007), \( 1.14\times10^{-5} \) to \( 6.09\times10^{-6} \) m\(^2\)/s for tomato pomace microwave drying at 160-800W (Alharahsheh, 2009), \( 0.55\times10^{-7} \) to \( 3.5\times10^{-7} \) m\(^2\)/s for Gundelia tournefortii microwave drying at 90-800W (Evin, 2011).
Figure 4. Comparison of moisture ratios determined by experimentation and prediction using the Midilli et al. model for carrot slices

Table 4. Values of effective diffusivity obtained for carrot slices at different microwave powers

<table>
<thead>
<tr>
<th>P(W)</th>
<th>Effective diffusivity (m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>4.20×10⁻⁷</td>
</tr>
<tr>
<td>300</td>
<td>3.56×10⁻⁶</td>
</tr>
<tr>
<td>500</td>
<td>6.60×10⁻⁶</td>
</tr>
<tr>
<td>700</td>
<td>7.34×10⁻⁶</td>
</tr>
</tbody>
</table>

The values of effective diffusivity versus m/P shown in Figure 5 accurately fit to Eq. (9) with coefficient of determination ($R^2$) of 0.9973. Then, $D_0$ and $E_a$ values were estimated as $1×10^{-5}$ m²/s and 92.33 W/g.

Figure 5. The relationship between the values of $D_{eff}$ versus sample amount/power

Figure 6 shows the microwave specific energy consumption values at different amounts of microwave powers for drying of carrot slices. Statistical analyses showed that microwave power was significant on the
specific energy consumption values of carrot slices (P ≤ 0.05). The values ranged from 10.27 to 23.29 kW×h/kg water evaporated.

CONCLUSION

Thin layer drying experiments were conducted to determine the thin layer drying characteristics and energy consumption of carrot slices in a microwave dryer. Five thin layer drying models were evaluated for their suitability. The Midilli et al. model was found to be the most suitable model for describing the thin layer drying of carrot slices. The effective moisture diffusivity was obtained based on Fick’s second law. The values ranged from 4.20×10^{-7} to 7.34×10^{-6} m²/s. The activation energy required to detach and move the water out from carrot slices during the drying process was found to be 92.33 W/g. Increasing the microwave power was caused to increase the drying rate and decrease the energy consumption. The values ranged from 10.27 to 23.29 kW×h/kg water evaporated.

REFERENCES